Solving 8x8 Classic Sudoku Puzzle Using Graph Coloring

D.Nithya, C.Shiny Climensa

Assistant Professor, Student

Department of Mathematics,

Hindusthan College of Arts and Science, Coimbatore, India

Abstract - Sudoku puzzles can be found in a variety of places, including newspapers, journals, mobile and restaurants. We will be given Sudoku to pass the time, and many of us will be perplexed as to how to solve the puzzle. Here is an easy way of solving Sudoku puzzle using graph coloring. By using graph coloring algorithm of 8x8 to find out Sudoku puzzle.

Index Terms – Graph coloring, proper coloring, chromatic number, 8x8 sudoku puzzle.

INTRODUCTION

Fill in the Sudoku board's 64 cells with numbers starting at 64 to complete the task. When this is the case, the graph related to a Sudoku puzzle is made up of 64 vertices (one for each cell on the board) and edges, where two vertices are connected by an edge if the cells they belong to are in the same column, row, or 2x4 box. As a result, we have graphed the Sudoku grid. If one wanted to draw this graph, it would have 64 vertices and several hundred edges, making it large. Instead, let's just consider this graph conceptually. How can we express the numbers that are found in the Sudoku puzzle's cell spaces? Give a colour to the numbers from 1 to 8. Now, change the vertices that correspond to a cell that contains a specific number to that number's colour.

Now it is clear that solving a sudoku without breaking the sudoku rule is comparable to colouring the vertices of the relevant graph while making sure that no two neighbouring vertices have the same colour.

ALGORITHM FOR GRAPH COLORING

Step 1: Arrange the graph's vertices in some order.

Step 2: Select the first vertex and fill it with the primary color.

Step 3: Pick the next vertex and color it with the lowest-numbered color that hasn't been used on any of the vertices nearby.

Step 4: If this color is used for all nearby vertices, change it to something else. Continue until all of the vertices have been coloured.

8x8 SUDOKU PUZZLE

An 8x8 Sudoku with 64 cells, 8 rows, and 8 columns that can be solved in the same way as a traditional 9x9 Sudoku with the exception of the irregular blocks. The 8x8 Sudoku resolution uses the same reflection processes as traditional Sudoku, but only has eight digits.

4				**1**			
				6		**4**	
	3						
7		**1**			**6**		
			7				**1**
5						**6**	
		7			**3**		
2			**8**				**5**

RULES OF 8X8 CLASSIC SUDOKU

Fill in each of the empty squares with a digit from 1 to 8, making sure that each digit appears exactly once in each row, column, and 2x4 outlined box.

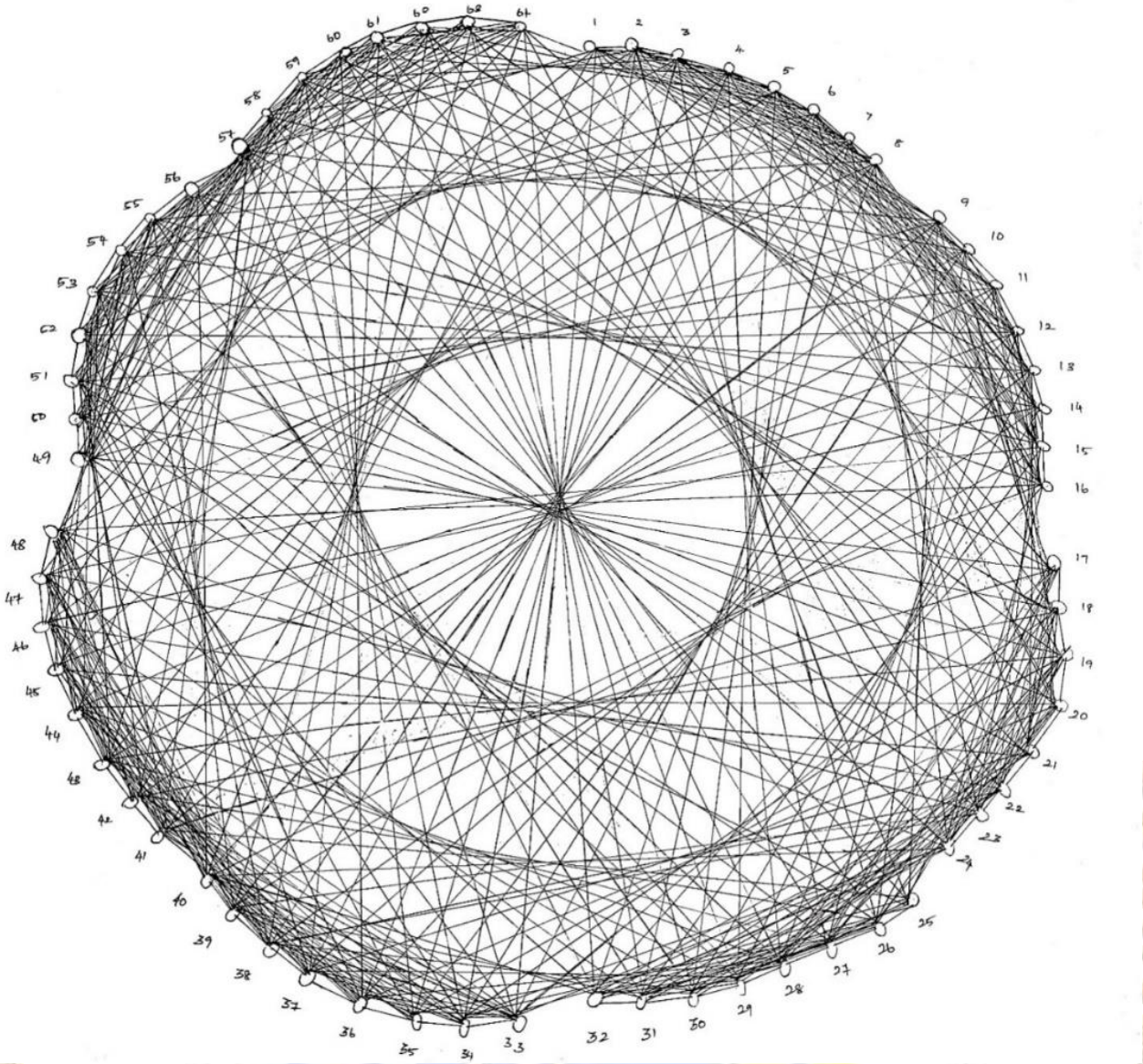
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
17	**18**	**19**	**20**	**21**	**22**	**23**	**24**
25	**26**	**27**	**28**	**29**	**30**	**31**	**32**
33	**34**	**35**	**36**	**37**	**38**	**39**	**40**
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**
49	**50**	**51**	**52**	**53**	**54**	**55**	**56**
57	**58**	**59**	**60**	**61**	**62**	**63**	**64**

Here there are 64 cells in the grid label the cells from 1 to 64 as this. To convert this problem into a graph plot 64 vertices with each vertex corresponding to a cell in the grid we need to draw the edges. Now we have to see the adjacency relationship If we give one number from 1 to 8 in one of the cells we cannot get the same number in the corresponding row the corresponding column and in the corresponding sub grid. So two distinct vertices will be adjacent if and only if the corresponding cells in the grid or either in the same sub grid.

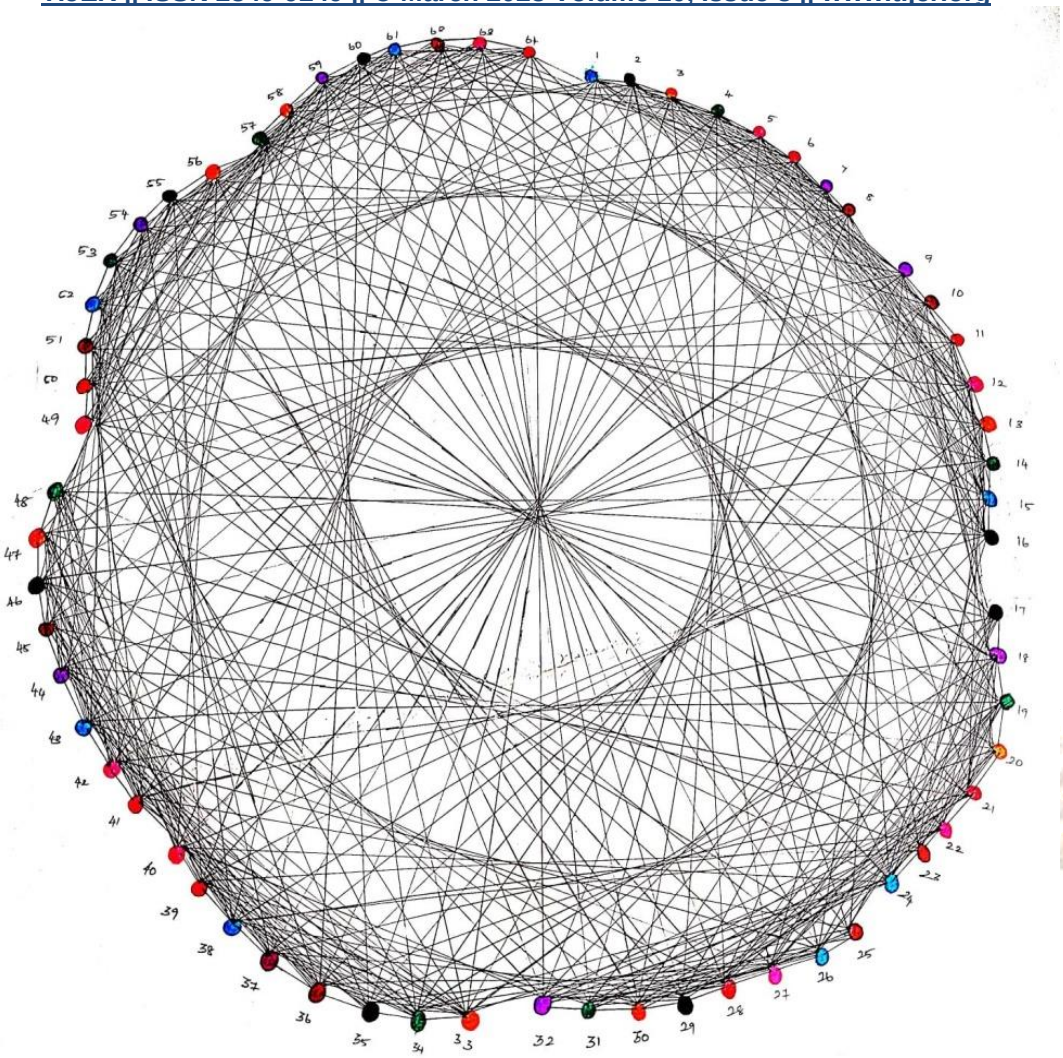
Now the vertex 1 is adjacent 2,3,4,5,6,7,8,9,17,25,33,41,49,57 and 11,12. The vertex 2 is adjacent 1,3,4,5,6,7,8,9,10,11,12,18,26,34,42 and 50,58. The vertex 3 is adjacent 1,2,4,5,6,7,8,9,10,11,12,19,27,35,43 and 51,59. The vertex 4 adjacent 1,2,3,5,6,7,8,9,10,11,12,20,28,36,44 and 52,60.

The vertex 5 adjacent 1,2,3,4,6,7,8,13,14,15,16,21,29,37,45 and 53,61.
The vertex 6 adjacent 1,2,3,4,5,7,8,13,14,15,16,212,30,38,46 and 54,62.
The vertex 7 adjacent 1,2,3,4,5,6,8,13,14,15,16,23,31,39,47 and 55,63.
The vertex 8 adjacent 1,2,3,4,5,6,7,13,14,15,16,24,32,40,48,56,64.

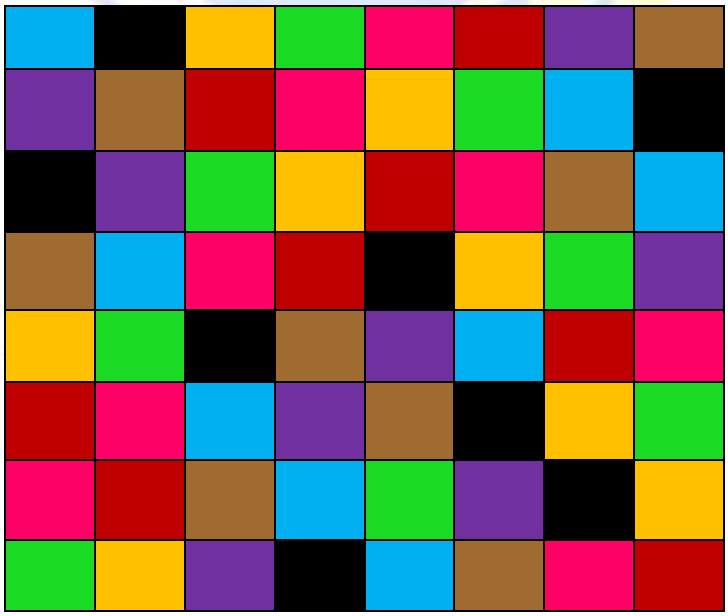
Continuing in this manner we will get the connected graph as given below.



Now we must colour the graph properly, which means using the fewest possible colours to colour the vertices of the graph so that no two neighbouring vertices have the same colour. In this graph, the minimal number of colours used is 8.



Color the cells as in the corresponding color of the vertex in the graph



We now give the digit 1 to any cell with the colour pink. Number 2 in the green cell. Number 3 in the purple cell. In blue, number four. 5 in the red-colored cells. Number 6 in the orange cell. Number 7 in the brown cells. Number 8 in the black cells.

4	8	6	2	1	5	3	7
3	7	5	1	6	2	4	8
8	3	2	6	5	1	7	4
7	4	1	5	8	6	2	3
6	2	8	7	3	4	5	1
5	1	4	3	7	8	6	2
1	5	7	4	2	3	8	6
2	6	3	8	4	7	1	5

CONCLUSION

As a result, we may claim that Sudoku can be regarded as a graph, and that it can be solved by colouring the graph with a chromatic number, G=8. It's the same as colouring the vertices with eight different colours so that no two neighbouring vertices are the same colour.

REFERENCES

[1].May, Katelyn D. (2020) "Colorings and Sudoku Puzzles," Rose-Hulman Undergraduate Mathematics Journal: Vol. 21 :Iss. 1 , Article 6.
[2].D. Marx, Graph coloring problems and their applications in scheduling, PeriodicaPolytechnica, Proc. John von Neumann PhD Students Conference, pp. 1–2, 2004.
[3] J. Schaad: Modeling the 8-queens problem and Sudoku using an algorithm based on projections onto nonconvex sets. Master's thesis, Univ. British Columbia (2010)
[4] Sander, Torsten (2009), "Sudoku graphs are integral", Electronic Journal of Combinatorics,
[5] R. Hesse, D.R. Luke: Nonconvex notions of regularity and convergence of fundamental algorithms for feasibility problems.
[6] F.T. Leighton: A graph coloring algorithm for large scheduling problems. J. Res.Nat. Bur. Standard 84(6) (1979), 489–506.
[7] V. G. Vizing: Coloring the vertices of a graph in prescribed colors. Diskret. Analiz.29 (1976), 3–10.
[8] P.M. Pardalos, T. Mavridou, J. Xue: The graph coloring problem: A bibliographic survey. In Handbook of combinatorial optimization, Springer US 1998, 1077–1141
[9] T.R. Jensen, B. Toft: Graph coloring problems. John Wiley & Sons, New York, 1995.