SOLVING 2*2 FUZZY GAME MATRIX PROBLEM USING OCTAGONAL, NONAGONAL FUZZY NUMBERS

[1]R.Umamaheswari
Research Scholar
Chikkanna Govt. Arts College, Tirupur.

[2]Dr.G.Sasikala
Assistant professor
Chikkanna Govt. Arts College, Tirupur.

Abstract:

In this paper, we consider a two player zero sum game with infinite value in pay-off matrix. Now we determine on solving a fuzzygame problem using octagonal, polygonal fuzzy number. To solve a problem using Pascal's triangular graded mean integration for finding a solution. We can convert a fuzzy valued game problem into a crisp valued game problem by changing the payoffs.

Keywords: Fuzzy set, Octagonal Fuzzy Number, Polygonal Fuzzy Number, Crisp Set, Pascal's Triangle Graded Mean Approach.

1. Introduction

Game theory has played an important role in the fields of decision making theory such as economics, management etc. This paper uses the concept of Pascal's triangle graded mean integration. It mainly uses octagonal fuzzy number, Polygonal fuzzy number to solve a fuzzy game problem.The defuzzified value can then be converted into a crisp value query, Which can then be answered using the conventional approach.

In this paper, First section includes some basic definition and second section includes mathematical formulation of fuzzy game problem and the third section includes algorithm for the problem and some numerical examples and in the final section we concludethe conclusion.

2. Preliminaries

In this section, we represent basic idea about the fuzzy numbers and definitions which involved in the research work.

Definition 1: A fuzzy set Ã in X (set of real numbers) is a set of ordered pairs Ã={(x, μ(Ã))/x ϵ X} is called membership function of x in Ã which maps X to[0,1].

Definition 2: A fuzzy collection A based on real numbers. If the membership function μ_A: R → [0, 1] of R has the following properties,it is said to be a fuzzy number.

(i)A is normal. It implies that X ∈R exists such that μ_A(x)=1.

(ii)A is convex. It implies that for every x_1, x_2 ∈R

$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min \mu_A(x_1),\ \mu_A(x_2), \lambda \in [0,1]$

(iii) μ_A is upper semi-continuous

(iv) Sup(A) is bounded in R.

Definition 3: A crisp set is a special case of a fuzzy set, in which the membershipfunction only takes two values, commonly defined as 0 and 1.

Definition 4: A Octagonal fuzzy number Ã = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) is a normal fuzzy number where a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 are real numbers and its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & for\ x < a_1 \\ h(\frac{x-a_1}{a_2-a_1}) & for\ a_1 \leq x \leq a_2 \\ k & for\ a_2 \leq x \leq a_3 \\ h+(1-h)(\frac{x-a_3}{a_4-a_3}) & for\ a_3 \leq x \leq a_4 \\ 1 & for\ a_4 \leq x \leq a_5 \\ h+(1-h)(\frac{a_6-x}{a_6-a_5}) & for\ a_5 \leq x \leq a_6 \\ h & for\ a_6 \leq x \leq a_7 \\ h(\frac{a_6-x}{a_6-a_7}) & for\ a_7 \leq x \leq a_8 \\ 0 & forx > a_8 \end{cases}$$

Where 0$< h < 1$

Definition 5: A Nonagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ is a normal fuzzy number where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ are real numbers and its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right) & for\ a_1 \le x \le a_2 \\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-a_2}{a_3-a_2}\right) & for\ a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \le x \le a_4 \\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-a_4}{a_5-a_4}\right) & for\ a_4 \le x \le a_5 \\ 1-\frac{1}{4}\left(\frac{x-a_5}{a_6-a_5}\right) & for\ a_5 \le x \le a_6 \\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) & for\ a_5 \le x \le a_6 \\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-a_7}{a_8-a_7}\right) & for\ a_7 \le x \le a_8 \\ \frac{1}{4}\left(\frac{a_9-x}{a_9-a_8}\right) & for\ a_8 \le x \le a_9 \\ o & otherwise \end{cases}$$

Figure 1: Pascal's Triangle Graded Mean Approach

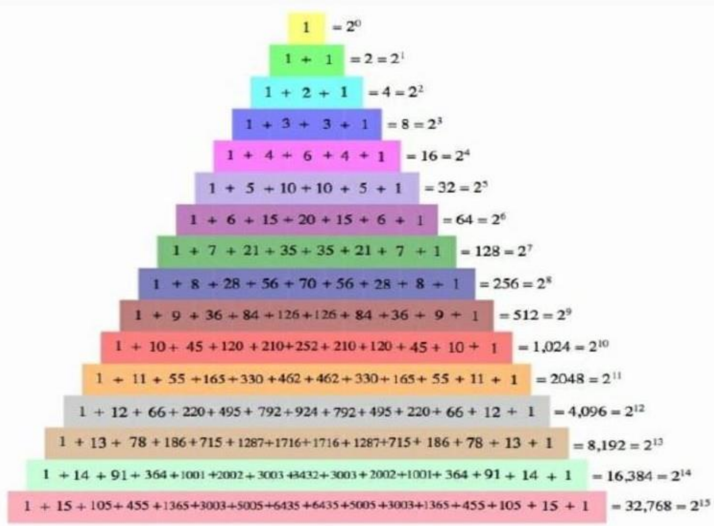


Figure: Pascal's Trapezoidal Approach

Let A= (a_1, a_2 ,a_3,a_4) be the trapezoidal fuzzy numbers then we can take the coefficient of fuzzy number from the pascal's triangle and apply the approach.

$$P(A)= \frac{x_1+7x_2+21x_3+35x_4+35x_5+21x_6+7x_7+x_8}{128}$$

The co-efficient x_1, x_2, x_3, x_4 are 1, 7,21,35,35,21,7, 1. This method can also be applied to order of n-dimensional Pascal.

Mathematical formulation of fuzzy game problem

Consider two competitors(called Players) in the game. Let Player A has m strategies A1, A2, . . .Am and Player B have n strategiesB1,B2, ..Bn. Here, it is assumed that each Player A has his choices from amongst the pure strategies .Assume that Player A is therefore considered to always be the winner and then the loser in any game is Player B. That is, in terms of Player A, all payoffs are presumed. If Player A selects approach Ai and Player B will choose strategy Bj . The payout matrix for player A follows.

Player B			
Player A	B_1	. . .	B_n
A_1	a_{11}	. . .	a_{1n}
	. . .. . .	. . .	. . .
A_m	a_{m1}	. . .	a_{mn}

Definition 6: Particular course of action that are selected by players, is called pure strategies.

Definition 7: Course of action that are to be selected on a particular occasion with some fixed probability are called mixedstrategies.

Definition 8: The maximin=minimax, the corresponding strategies which give the saddle point are called optimal strategies.

Definition 9: Rectangular games with two teams are called zero-sum games. In this case, one player's loss (gain) is exactlyequal to the other player's gain (loss).

Definition 10: If the maximin value equals the minimax value, then the game is said to have a saddle point.

Definition 11: A zero-sum game is one in which the algebraic sum of all player's profits and losses equals zero. The game isotherwise called a non-zero-sum-game.

Algorithm for solving a problem

Step 1: By using Pascal's Triangle Graded Mean Approach .We can convert given octagonal fuzzy numbers to a crisp values.

Step 2: If the saddle point exist we obtain a value of the game. If saddle point does not exist go to next step.

Step 3: Optimum Mixed strategy $S_A=\begin{pmatrix} A_1 & A_2 \\ p_1 & p_2 \end{pmatrix}$, $S_B=\begin{pmatrix} B_1 & B_2 \\ q_1 & q_2 \end{pmatrix}$

$P_1=\frac{a_{22}-a_{21}}{\lambda}$

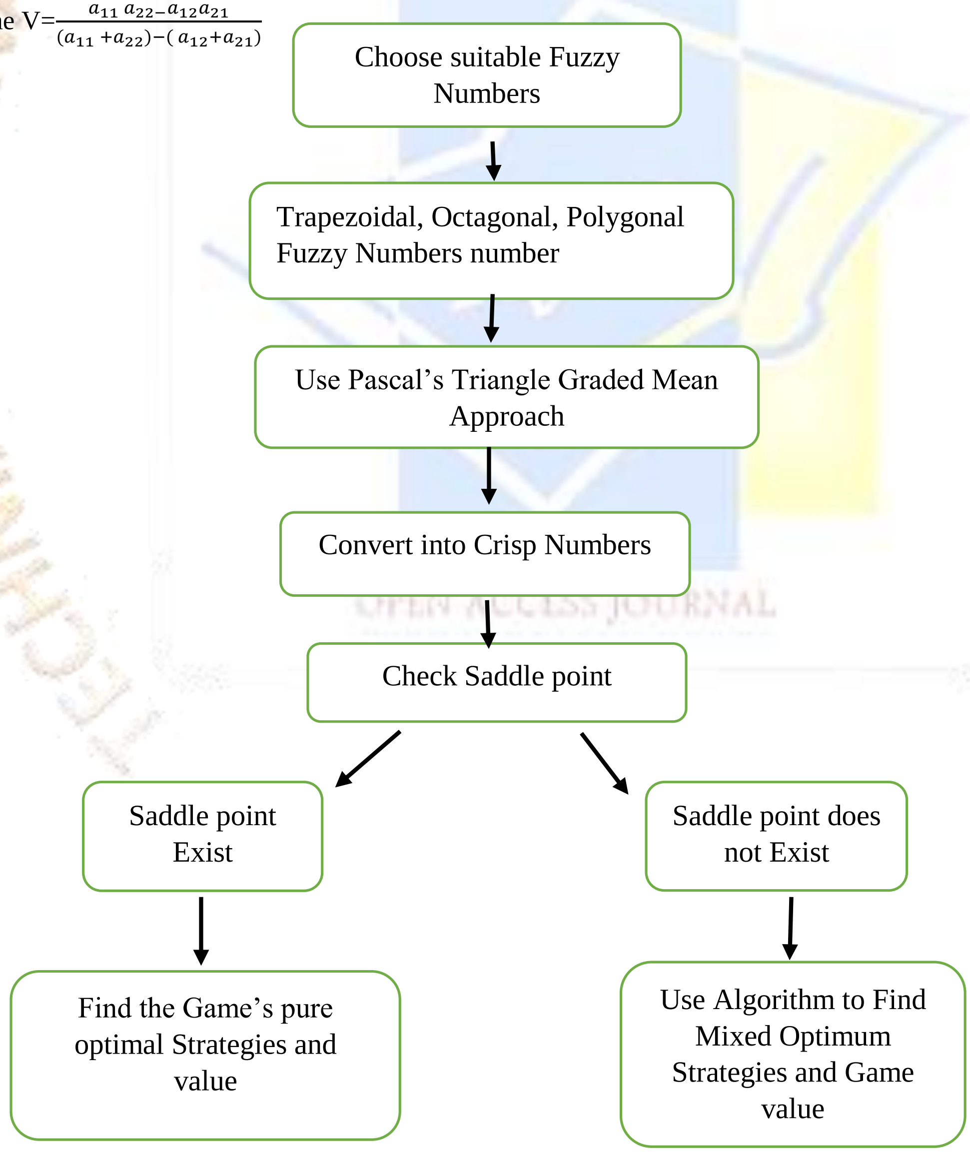
$P_2 = 1-P_1$

$q_1 = \frac{a_{22}-a_{12}}{\lambda}$

$q_2=1-q_1$

$\lambda=(a_{11}+a_{22})-(a_{12}+a_{21})$

Value of the game $V=\frac{a_{11}a_{22}-a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$



Numerical Example

Consider the following fuzzy game problem

Player B

$$\text{Player A}\begin{pmatrix}(-3,-1,0,1,2,3,4,5) & (1,2,3,4,5,6,7,8)\\(2,3,4,5,6,7,8,9) & (-3,-1,3,6,8,9,10,12)\end{pmatrix}$$

Solution

Step 1: Use Pascal's Graded Mean Approach to convert a given fuzzy problem into a crisp value(Table)

$$P(A)=\frac{x_1+7x_2+21x_3+35x_4+35x_5+21x_6+7x_7+x_8}{128}$$

Step 2: The payoff matrix

Player B

$$\text{Player A}\begin{pmatrix}1.492 & 4.5\\5.546 & 6.359\end{pmatrix}$$

Minimum value of the first row = 1.492,Minimum value of the second row =5.546

Maximum value of the first column =5.546,Maximum value of the second column =6.359

Maximin value = maximum(minimum value) = maximum(1.492,5.546) =5.546

Minimax value = minimum(maximum value) = minimum(5.546,6.359) = 5.546

Minimax = Maximim

Saddle point is (5.546,5.546)

Therefore value of the game is V=5.546.

Therefore the game is said to be strictly determinable.

Table

Crisp value for the fuzzy problem

a_{11}=$(-3,-1,0,1,2,3,4,5)$	P(a_{11}) =$\frac{-3+7(-1)+21(0)+35(1)+35(2)+21(3)+7(4)+5}{128}$ = 1.492
a_{12}=$(1,2,3,4,5,6,7,8)$	P(a_{12}) =$\frac{1+7(2)+21(3)+35(4)+35(5)+21(6)+7(7)+8}{128}$ = 4.5
a_{21}=$(2,3,4,5,6,7,8,9)$	P(a_{21}) =$\frac{2+7(3)+21(4)+35(5)+35(6)+21(7)+7(8)+9}{128}$ = 5.546
a_{22}=$(-3,-1,3,6,8,9,10,12)$	P(a_{22}) =$\frac{-3+7(-1)+21(3)+35(6)+35(8)+21(9)+7(10)+12}{128}$ = 6.356

Numerical Example

Consider the following fuzzy game problem

Player B

$$\text{Player A}\begin{pmatrix}(-3,-2,1,3,6,8,10,13) & (-1,0,1,2,3,5,8,12)\\(-5,-3,-2,1,2,3,4,5) & (1,2,3,4,5,6,10,18)\end{pmatrix}$$

Solution

Step 1: Use Pascal's Graded Mean Approach to convert a given fuzzy problem into a crisp value(Table)

$$P(A)=\frac{x_1+7x_2+21x_3+35x_4+35x_5+21x_6+7x_7+x_8}{128}$$

Step 2: The payoff matrix

Player B

Player A $\begin{pmatrix} 4.453 & 2.875 \\ 1.04 & 4.742 \end{pmatrix}$

Minimum value of the first row = 2.875

Minimum value of the second row =1.04

Maximum value of the first column =4.453

Maximum value of the second column =4.742

Maximin value = maximum(minimum value) = maximum(2.875,1.04) =2.875

Minimax value = minimum(maximum value) = minimum(4.453,4.742) = 4.453

Minimax ≠ Maximim

It does not exist saddle point.

Table

Crisp value for the fuzzy problem

$a_{11}=(-3,-2,1,3,6,8,10,13)$	$P(a_{11})=\frac{-3+7(-2)+21(1)+35(3)+35(6)+21(8)+7(10)+13}{128}=4.453$
$a_{12}=(-1,0,1,2,3,5,8,12)$	$P(a_{12})=\frac{-1+7(0)+21(1)+35(2)+35(3)+21(5)+7(8)+12}{128}=2.285$
$a_{21}=(-5,-3,-2,1,2,3,4,5)$	$P(a_{21})=\frac{-5+7(-3)+21(-2)+35(1)+35(2)+21(3)+7(4)+5}{128}=1.04$
$a_{22}=(1,2,3,4,5,6,10,18)$	$P(a_{22})=\frac{1+7(2)+21(3)+35(4)+35(5)+21(6)+7(10)+18}{128}=4.742$

Step 3: To find the optimum mixed strategy and value of the game.

Here $\boldsymbol{a_{11}}$ =4.453, $\boldsymbol{a_{12}}$=2.875, $\boldsymbol{a_{21}}$=1.04, $\boldsymbol{a_{22}}$=4.742

$\lambda=(a_{11}+a_{22})-(a_{12}+a_{21})$ = (4.453+4.742)-(2.875+1.04) =5.28

$p_1=\frac{\boldsymbol{a_{22}-a_{21}}}{\lambda}=\frac{4.742-1.04}{5.28}=0.701$

p_2 =1-p_1=1-0.701=0.299

$q_1=\frac{\boldsymbol{a_{22}-a_{12}}}{\lambda}=\frac{4.742-2.875}{5.28}=0.353$

q_2=1-q_1 =1-0.353=0.647

Value of the game $V=\frac{a_{11}a_{22}-a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}=\frac{4.453(4.742)-(2.875)1.04}{(4.453+4.742)-(2.875+1.04)}=3.433$

Value of the game V = 3.433.

Numerical Example

Consider the following fuzzy game problem

Player B

$$\text{Player A}\begin{pmatrix}(-3,-1,0,2,3,6,7,8,9) & (-4,-3,-1,0,1,2,4,5,6)\\(-2,-1,2,3,4,6,8,9,10) & (0,1,2,3,4,5,6,7,8)\end{pmatrix}$$

Solution

Step 1: Use Pascal's Graded Mean Approach to convert a given fuzzy problem into a crisp value(Table)

$$P(A)=\frac{x_1+8x_2+28x_3+56x_4+70x_5+56x_6+28x_7+8x_8+x_9}{256}$$

Step 2: The payoff matrix

Player B

$$\text{Player A}\begin{pmatrix}1.109 & 3.578\\4.437 & 4\end{pmatrix}$$

Minimum value of the first row = 1.109

Minimum value of the second row =4

Maximum value of the first column =4.437

Maximum value of the second column =4

Maximin value = maximum(minimum value) = maximum(1.109,4) =4

Minimax value = minimum(maximum value) = minimum(4.437,4) = 4

Minimax = Maximim

Saddle point is (4,4)

Therefore value of the game is V=4

Therefore the game is said to be strictly determinable.

Table

Crisp value for the fuzzy problem

$a_{11}=(-3,-1,0,2,3,6,7,8,9)$	$P(a_{11})=\frac{-3+8(-1)+28(0)+56(2)+70(3)+56(6)+28(7)+8(8)+9}{256}=1.109$
$a_{12}=(-4,-3,-1,0,1,2,4,5,6)$	$P(a_{12})=\frac{-4+8(-3)+28(-1)+56(0)+70(1)+56(2)+28(4)+8(5)+6}{256}=3.578$
$a_{21}=(-2,-1,2,3,4,6,8,9,10)$	$P(a_{21})=\frac{-2+8(-1)+28(2)+56(3)+70(4)+56(6)+28(8)+8(9)+10}{256}=4.437$
$a_{22}=(0,1,2,3,4,5,6,7,8)$	$P(a_{22})=\frac{0+8(1)+28(2)+56(3)+70(4)+56(5)+28(6)+8(7)+8}{256}=4$

Numerical Example

Consider the following fuzzy game problem

Player B

$$\text{Player A}\begin{pmatrix}(-3,-2,1,3,6,8,10,13,14) & (-1,0,1,2,3,5,8,12,13)\\(-5,-3,-2,1,2,3,4,5,6) & (1,2,3,4,5,6,10,18,19)\end{pmatrix}$$

Solution

Step 1: Use Pascal's Graded Mean Approach to convert a given fuzzy problem into a crisp value(Table)

$$P(A)=\frac{x_1+8x_2+28x_3+56x_4+70x_5+56x_6+28x_7+8x_8+x_9}{256}$$

Step 2: The payoff matrix

Player B

Player A $\begin{pmatrix} 5.636 & 3.757 \\ 1.707 & 5.679 \end{pmatrix}$

Minimum value of the first row = 3.757

Minimum value of the second row =1.707

Maximum value of the first column =5.636

Maximum value of the second column =5.679

Maximin value = maximum(minimum value) = maximum(3.757,1.707) =3.757

Minimax value = minimum(maximum value) = minimum(5.636,5.679) = 5.636

Minimax ≠ Maximim

It does not exist saddle point.

Table

Crisp value for the fuzzy problem

a_{11}=(−3, −2,1,3,6,8,10,13,14)	P(a_{11}) $=\frac{-3+8(-2)+28(1)+56(3)+70(6)+56(8)+28(10)+8(13)+14}{256}$ = 5.636
a_{12}= (−1,0,1,2,3,5,8,12,13)	P(a_{12}) $=\frac{-1+8(0)+28(1)+56(2)+70(3)+56(5)+28(8)+8(12)+13}{256}$ = 3.757
a_{21}=(−5, −3, −2,1,2,3,4,5,6)	P(a_{21}) $=\frac{-5+8(-3)+28(-2)+56(1)+70(2)+56(3)+28(4)+8(5)+6}{256}$ = 1.707
a_{22}=(1,2,3,4,5,6,10,18,19)	P(a_{22}) $=\frac{1+8(2)+28(3)+56(4)+70(5)+56(6)+28(10)+8(18)+19}{256}$ = 5.679

Step 3: To find the optimum mixed strategy and value of the game.

Here $\boldsymbol{a_{11}}$ =5.636, $\boldsymbol{a_{12}}$=3.757, $\boldsymbol{a_{21}}$=1.707, $\boldsymbol{a_{22}}$=5.679

λ=(a_{11} +a_{22})-(a_{12} + a_{21}) = (5.636+5.679)-(3.757+1.707) =5.851.

$p_1=\frac{\boldsymbol{a_{22}-a_{21}}}{\lambda}=\frac{5.679-1.707}{5.851}$=0.678.

p_2 =1-p_1=1-0.678=0.322.

$q_1=\frac{\boldsymbol{a_{22}-a_{12}}}{\lambda}=\frac{5.679-3.757}{5.851}$ = 0.328.

q_2=1-q_1 =1-0.328=0.672.

Value of the game V=$\frac{a_{11}\,a_{22}-a_{12}a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}=\frac{5.636(5.679)-3.757(1.707)}{(5.636+5.679)-(3.757+1.707)}$ = 4.374.

Value of the game V = 4.374.

Conclusion

In this paper, solving a fuzzy game problem using Pascal's Triangle Graded Mean Approach. To solve any 2*2 Matrix with its values as Polygonal fuzzy number. These values can be converted to crisp values using Pascal's Triangle Graded Mean Approach. It will be useful in the future problems involving octagonal fuzzy numbers.

References

[1] K.Sangeetha and M.Parimala, on solving a fuzzy game problem using hexagonal fuzzy numbers,2021.

[2] L.A Zadeh, Fuzzy sets, Information and control,vol 8, 1965,338-353.

[3] M.Thirucheran,E.R Meena kumari,S.Lavanys, A new approach for solving fuzzy game problem,vol 114,2017,67-75.

[4] K.Selvakumari and S.Lavanya, "on solving fuzzy game problem using octagonal fuzzy numbers",Annals of pure and applied mathematics,8(2),2014,211-217.

[5] G.krishnaveni and k.Ganesan, A new approach for the solution of fuzzy games,j.phys conf.ser,2018.

[6] A.Thamariselvi, R.Santhi, Optimal solution of fuzzy transportation problem using hexagonal fuzzy numbers,Int.j.science.eng.res 6(2015),2229-5518.

[7] P.Rajarajeshwari, A.Sahaya sudha, Ordering generalised hexagonal fuzzy number using rank, mode,divergence and spreadIOSR J.Mathematics(2014).

[8] S.U.Malini, C.K.Felbin, An approach for solving fuzzy transportation problem using octagonal fuzzy numbers, Applied Mathematical Sciences,7(54)(2013),2661-2673.

[9] R.Senthil Kumar and S.Kumaraghuru,"Solution of fuzzy game problem using triangular fuzzy number" IJISET,2(2),2015.

[10] K.Selvakumari and S.Lavanya, "An approach for solving fuzzy game problem", Indian journal of science and Technology,8(15),2015.

[11] C.R.Bector, S.Chandra, Fuzzy Mathematical Programming and Fuzzy Matrix Games, Springer verlag ,New york,2005.

[12] D.S.Dinagar, U.H. Naryanan ,On determinant of hexagonal fuzzy number matrices", Int.J.Math.Appl.4,2016.

[13] V.K kapoor ,S.Kapoor "Operation Research Techniques for management"Edition 7.S.chand.2001.

[14] Bompard E, Carpaneto E, Ciwei G, Napoli R, Benini M, Gallanti Mand Migliavacca G,2008 Agame theory simulator for assessing the performances of competitive electricity markets *Electric Power Systems Research* **78 (2)** pp 217- 227.

[15] Campos L, Fuzzy linear programming models to solve fuzzy matrix games, 1989 *Fuzzy Sets andSystems*, **32(3)**pp 275-289.

[16] Charilas D E, and Panagopoulos A D, 2010 A survey on game theory applications in wirelessnetworks *Computer Networks*, **54 (18)** pp 3421- 3430.

[17] Fiestras-Janeiro M G, Garcia-Jurado I, Meca A and Mosquera M A, 2011 Cooperative gametheory and inventory management *European Journal of Operational Research* , **210 (3)** pp 459- 466.

[18] Madani K, 2010 Game theory and water resources *Journal of Hydrology*, **381 (3-4)**pp 225-238.

[19] Sakawa M and Nishizaki I, 1994 Max-min solution for fuzzy multi-objective matrix games *Fuzzy Sets and Systems*, **67(1)** pp 53-59

[20] S.Divya Bhrathi and P.Saraswathi-IJME Vol.7 No.4 April,2022 "*Fuzzy Game Problem Using octagonal Fuzzy Numbers"*

[21] Von Neumann J and Morgenstern D,1944 Theory of Games in Economic Bahavior, New York,Wiley .